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ELASTIC WAVE PROPAGATION IN A POROUS LAMINATED COMPOSITE†

C. SVE‡

The Aerospace Corporation, El Segundo, California, U.S.A.

Abstract—An analysis is presented for linear elastic wave propagation normal to the laminations of a periodically laminated porous composite. The porosity is randomly distributed throughout one constituent and is composed of small spherical voids. This type of porosity is called randomly periodic and produces Rayleigh scattering where the wavelength of the incident wave is much larger than the void diameter. Porosity also reduces the wave speed in a constituent and thereby affects geometric dispersion. A dissipative equation of motion is developed for porous material that includes a porosity-dependent wave speed and a scattering term that provides spatial attenuation. This equation is then used for a constituent of a composite, and a dispersion relation and pulse solution are obtained to determine the significance of porosity in a laminated composite. It is concluded that Rayleigh scattering produces a small damping effect in far-field pulse shapes and small-void porosity can be adequately simulated with an effective wave speed.

1. INTRODUCTION

ELASTIC wave propagation in composite materials is influenced by a phenomenon called geometric dispersion where the phase velocity of harmonic waves depends on their frequency. This tends to spread out a pulse as it travels through a composite. In this paper a model is presented that describes a medium that is not only geometrically dispersive by virtue of being periodically laminated, but also alters a pulse form because of the porosity of one of its constituents. Previous work has dealt with nonporous laminated media, effective moduli for porous material, and scattering from spherical cavities. This model attempts to incorporate all of these results to assess the importance of porosity in composite materials.

Geometric dispersion in nonporous laminated materials has been studied by Rytov [1], Sun, Achenbach and Herrmann [2] and Sve [3] and is based on the equations of linear elasticity. It was found that the frequency spectrum for waves propagating across the laminations was periodic in wave number and consisted of an infinite number of modes with stop and pass bands. Sun, Achenbach and Herrmann developed an effective stiffness theory that compared well with the exact results for long wavelengths and low frequencies.

When porosity is added to a material, the density and elastic modulus decrease. Mackenzie [4] determined effective elastic moduli for a slightly porous material that contains a random distribution of small spherical holes. Hashin [5] has considered the problem of a finite concentration of spherical elastic inclusions in an elastic medium, and obtained bounds for the moduli. These results are used in the present paper to estimate an effective wave speed for a porous medium.

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‡ Member of the Technical Staff, Mechanics Research Department.

Scattering occurs when a wave encounters an inclusion, and these problems have been treated in considerable detail. Yamakawa [6, 7] reviews scattering results for various kinds of spherical obstacles including cavities, and calculates the energy decrease for waves traveling through a medium containing N spherical obstacles per unit volume. Ying and Truell [8] presented scattering cross sections for various spherical inclusions in the Rayleigh region. Knopoff (9, 10] presented compression and shear wave scattering calculations for an isolated rigid sphere in an elastic medium. Scattering of transverse waves by a spherical obstacle was addressed by Einspruch *et al.* [11]. Pao and Mow [12] examined the case investigated by Ying and Truell and included the rigid body motion for the rigid inclusion. An effective continuum theory that included the effect of scattering for low frequency was developed by Moon and Mow [13] for wave propagation in a material composed of small rigid spherical inclusions. Their results indicated that an incident pulse is spread out by geometric dispersion and is also attenuated due to scattering. They determined that the attenuation depended on the fourth power of the frequency, which is called radiation damping. Moon and Mow also verified that this attenuation was proportional to the scattering cross section of a single inclusion. Scattering of elastic waves by fibers has been considered by Mok [14].

The approach of the present paper is to propose an equation of motion that reproduces the known results for a medium composed of a few spherical holes that are randomly distributed. Dispersion and attenuation relations are presented for this equation to verify that it has the proper behavior in the low frequency region. The result is a nondispersive wave with a wave speed that decreases with porosity. The wave is also spatially attenuated where the attenuation is proportional to the volume of a void and to the fourth power of the frequency. This equation is then used for one of the constituents of a laminated composite to determine the effect of porosity on the dispersion and attenuation properties of a composite. Analytical results are presented for the wave speed, dispersion and attenuation parameters, and pulse shape in a porous laminated composite. Several numerical examples are included that illustrate the interaction of porosity with the geometric dispersion due to the periodicity of the composite.

2. POROUS MATERIAL

Knopoff [15] and Newlands [16] have dealt with dissipative equations of motion in the simulation of viscoelastic response. In the present case, a dissipative equation is used that reproduces the low-frequency portion of the exact results obtained by Ying and Truell [8] for scattering. This does not mean that scattering of energy by inclusions is caused by a viscoelastic mechanism. When scattering occurs, some energy is taken from the incident wave and scattered in all directions, which contributes to the attenuation or extinction of the incident wave as it progresses through the medium. This scattered energy is not necessarily converted into heat or lost. It is simply delayed or redirected, and the total energy is conserved.

The basic equation used in this paper to simulate the average low-frequency dynamic behavior of a porous material is

$$(\lambda+2\mu)u_{,xx}-\eta u_{,xxtt} = \rho u_{,tt} \tag{1}$$

where u is average displacement, t is time, ρ is effective density, x is a spatial coordinate, and η is a coefficient (to be defined later) that provides dissipation. The following development will provide the motivation for this equation.

The effective Lamé parameters, λ and μ, are related to the nonporous parameters λ_0 and μ_0. Mackenzie's [4] result for the effective bulk modulus K in terms of the porosity p and the nonporous Poisson's ratio v_0 is

$$\frac{K}{K_0} = 1 - \frac{3(1-v_0)p}{2(1-2v_0)+(1+v_0)p} \tag{2}$$

and an estimate of the effective shear modulus from Hashin's [5] analysis is

$$\frac{\mu}{\mu_0} = 1 - \frac{15(1-v_0)p}{7-5v_0+2(4-5v_0)p}. \tag{3}$$

This leads to an effective wave speed

$$v_d = \left(\frac{\lambda+2\mu}{\rho}\right)^{\frac{1}{2}} = v_0\left(\frac{g_0}{1-p}\right)^{\frac{1}{2}} \tag{4}$$

where the nonporous wave speed is

$$v_0 = \left(\frac{\lambda_0+2\mu_0}{\rho_0}\right)^{\frac{1}{2}} \tag{5}$$

and the effective density is

$$\rho = \rho_0(1-p). \tag{6}$$

The function g_0 depends only on the values of porosity and nonporous Poisson's ratio and is given by

$$g_0 = \frac{1-2v_0}{1-2v_0+\frac{p}{2}(1+v_0)} - \frac{10p(1-2v_0)}{7-5v_0+2(4-5v_0)p}. \tag{7}$$

These results are for a porous material that contains a small concentration of spherical cavities and should not be considered valid for all values of porosity. (The reader is referred to Hashin [5] for a discussion of finite concentrations.) The dependence of the reduced wave speed on the porosity and v_0 is illustrated in Fig. 1. As the nonporous material becomes incompressible, v_0 approaches 0·5 and v_d/v_0 goes to zero since v_0 approaches infinity and v_d remains finite.

In order to relate equation (1) further to porous media response, a traveling wave solution is developed, i.e.

$$u \sim \exp[i(\kappa x - \omega t)]. \tag{8}$$

Substitution of equation (8) into (1) leads to a relation between the wave number κ and the frequency ω, which is

$$\kappa^2 = \frac{\omega^2}{v_d^2(1-i\varepsilon\omega^3)} \tag{9}$$

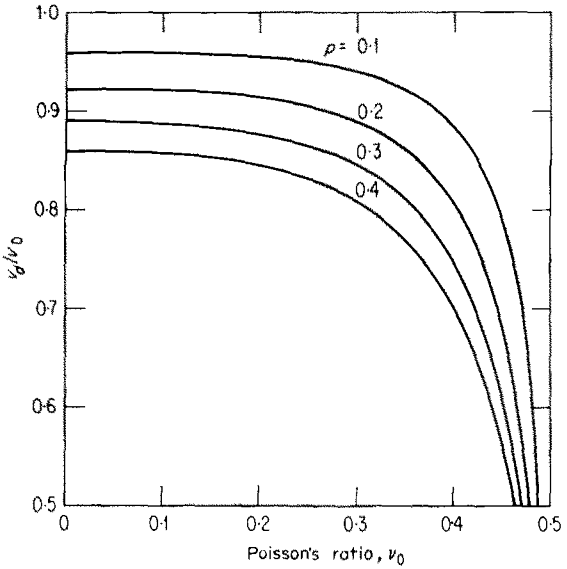

Fig. 1. Effective wave speed vs. Poisson's ratio.

where

$$\varepsilon = \eta/(\lambda+2\mu). \tag{10}$$

The separation of κ into its real and imaginary parts yields

$$\kappa_r^2 = \frac{\omega^2[1+(1+\varepsilon^2\omega^6)^{\frac{1}{2}}]}{2v_d^2(1+\varepsilon^2\omega^6)} \tag{11}$$

$$\kappa_i^2 = \frac{\omega^2[-1+(1+\varepsilon^2\omega^6)^{\frac{1}{2}}]}{2v_d^2(1+\varepsilon^2\omega^6)}. \tag{12}$$

Since the low-frequency range is of interest in this study, we expand equations (11 and 12) to give

$$\kappa_r \simeq \omega/v_d, \qquad \kappa_i \simeq \varepsilon\omega^4/2v_d \tag{13}$$

when $\varepsilon^2\omega^6 \ll 1$, which yields a nondispersive wave that is spatially attenuated.

If equation (1) is to be useful in the low-frequency response regime, it must be related to the known results of Ying and Truell [8]. In their work for a single scatterer, they presented a low-frequency expression for the scattering cross-section for a small spherical cavity of radius a. Their result in the present notation is

$$S_c = 4\pi g_c a^6(\omega/v_0)^4/9 \tag{14}$$

where

$$g_c = 4/3+(80+120\delta^5)/(4-9\delta^2)^2-3\delta^2/2+2\delta^3/3+9\delta^4/16, \qquad \delta^2 = 2(1-\nu_0)/(1-2\nu_0). \tag{15}$$

The dependence of g_c on Poisson's ratio ν_0 is shown in Fig. 2. In order to obtain the attenuation of N scatterers per volume, an assumption is required concerning the influence of a scatterer on its neighbors. If they are independent and randomly distributed throughout the material,

$$\kappa_i = NS_c/2 = pg_c a^3(\omega/v_0)^4/6 \tag{16}$$

where the porosity is

$$p = 4\pi a^3 N/3. \tag{17}$$

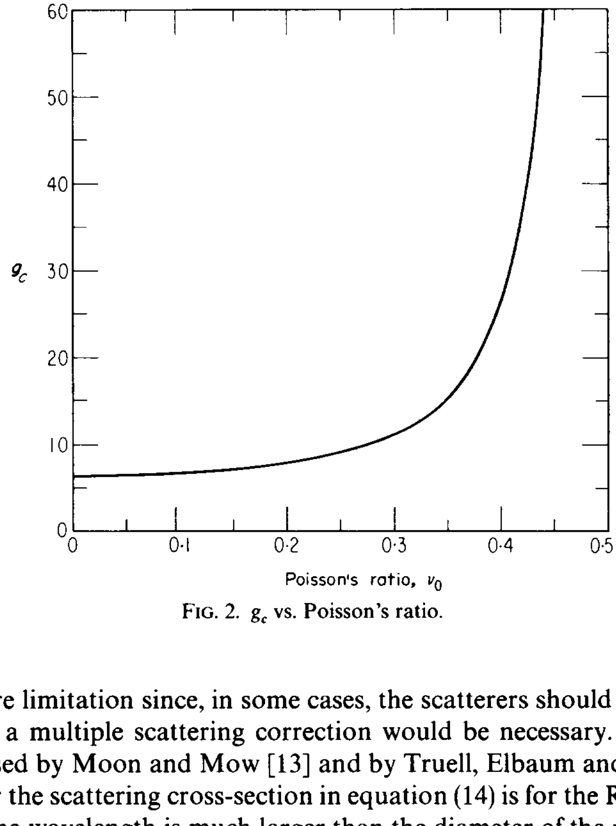

FIG. 2. g_c vs. Poisson's ratio.

This is a severe limitation since, in some cases, the scatterers should not be considered independent and a multiple scattering correction would be necessary. These limitations have been discussed by Moon and Mow [13] and by Truell, Elbaum and Chick [17]. Also, the expression for the scattering cross-section in equation (14) is for the Rayleigh scattering region in which the wavelength is much larger than the diameter of the void. In any event, for a dilute mixture of small random voids, it is now possible to compare equations (13 and 16) in order to determine an explicit expression for ε so that the proposed equation of motion (1) can be used for one or more of the constituents of a composite. The result is

$$\varepsilon = \frac{pg_c a^3 v_d}{3v_0^4}. \tag{18}$$

Since an expression for ε is now available, dispersion and attenuation curves can be developed from equations (11 and 12). The effect of Rayleigh-type porosity on the real part of the wave number for $\nu_0 = 0{\cdot}3$ is shown in Fig. 3. In the low-frequency range, the real part of the wave number is linear in frequency, which indicates a nondispersive result. The behavior of the imaginary part of the wave number for increasing amounts of porosity is illustrated in Fig. 4. For small frequencies in agreement with scattering results, the imaginary part of the wave number is proportional to the fourth power of the frequency.

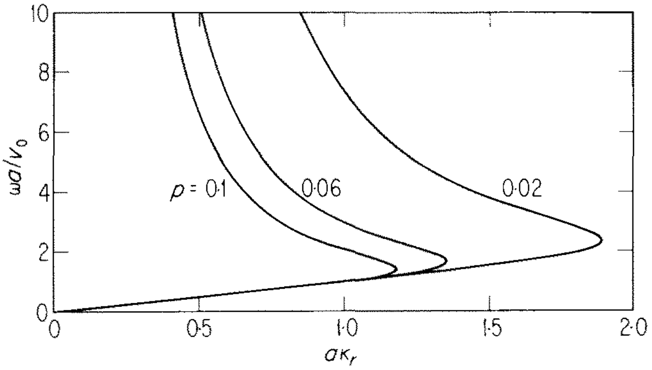

Fig. 3. Frequency vs. real part of wave number for porous medium.

3. POROUS LAMINATED COMPOSITE

An analysis based on this model for a porous composite is presented to illustrate the interaction of geometric dispersion and porosity. In this periodic laminated composite (Fig. 5), one constituent contains a dilute random distribution of spherical voids. Both constituents are otherwise linear elastic, isotropic, homogeneous materials where the equations of motion are

$$(\lambda_1+2\mu_1)u_{,xx}-\eta_1 u_{,xxtt} = \rho_1 u_{,tt}, \quad \text{for layer 1} \tag{19}$$

$$(\lambda_2+2\mu_2)u_{,xx} = \rho_2 u_{,tt}, \quad \text{for layer 2} \tag{20}$$

where u is the displacement normal to the laminations and λ_1, μ_1, ρ_1 are the effective constants that depend on λ_0, μ_0, ρ_0 and p_1, the porosity in layer 1.

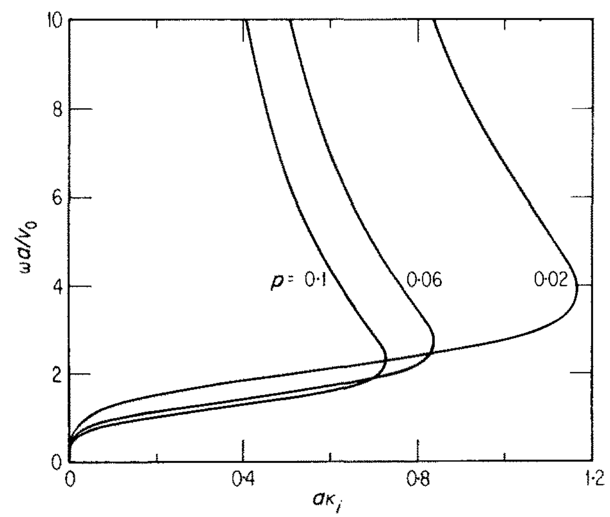

Fig. 4. Frequency vs. imaginary part of wave number for porous medium.

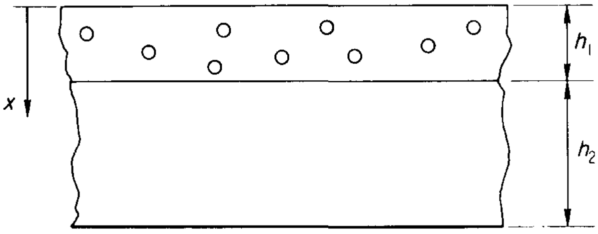

FIG. 5. Porous laminated composite.

A traveling wave solution is developed with the technique used by Rytov [1] for the nonporous case. The displacement is assumed to be of the form

$$u = f(x)\exp[i(\kappa x - \omega t)]. \tag{21}$$

Bloch's theorem (see Brillouin [18]) requires that $f(x)$ have the same periodicity as the composite. Continuity of the displacement and stress at the interfaces must also be satisfied by the solution.

Substitution of equation (21) into (19 and 20) yields two ordinary differential equations with solutions

$$f(x) = A_1 \exp[i(-\kappa + \omega/\tilde{c}_1)x] + A_2 \exp[i(-\kappa - \omega/\tilde{c}_1)x] \tag{22}$$

for layer 1, and

$$f(x) = B_1 \exp[i(-\kappa + \omega/c_2)x] + B_2 \exp[i(-\kappa - \omega/c_2)x] \tag{23}$$

for layer 2, in which

$$\tilde{c}_1 = c_1(1 - i\varepsilon_1\omega^3)^{\frac{1}{2}}, \qquad c_2 = \left(\frac{\lambda_2 + 2\mu_2}{\rho_2}\right)^{\frac{1}{2}} \tag{24}$$

where

$$c_1 = v_0\left(\frac{g_0}{1-p_1}\right)^{\frac{1}{2}}, \qquad \varepsilon_1 = \frac{\eta_1}{(\lambda_1 + 2\mu_1)} = \frac{p_1 g_c c_1 a^3}{3v_0^4}. \tag{25}$$

The matching and periodicity conditions require

$$f(0^+) = f(0^-) \tag{26}$$

$$f(h_1^-) = f(-h_2^+) \tag{27}$$

$$(1 - i\varepsilon_1\omega^3)(f' + i\kappa f)|_{x=0^+} = \gamma(f' + i\kappa f)|_{x=0^-} \tag{28}$$

$$(1 - i\varepsilon_1\omega^3)(f' + i\kappa f)|_{x=h_1^-} = \gamma(f' + i\kappa f)|_{x=-h_2^+} \tag{29}$$

where

$$\gamma = \frac{\lambda_2 + 2\mu_2}{\lambda_1 + 2\mu_1} = \frac{\lambda_2 + 2\mu_2}{(\lambda_0 + 2\mu_0)g_0}. \tag{30}$$

Substitution of equations (22 and 23) into (26–29) leads to four homogeneous algebraic equations. For a nontrivial solution, the following determinant must be zero:

$$\begin{vmatrix} 1 & 1 & -1 & -1 \\ e_1 & e_2 & -e_3 & -e_4 \\ s & -s & -1 & 1 \\ se_1 & -se_2 & -e_3 & e_4 \end{vmatrix}. \tag{31}$$

In this determinant,

$$\begin{aligned} e_1 &= \exp[ih_1(-\kappa+\omega/\tilde{c}_1)] \\ e_2 &= \exp[ih_1(-\kappa-\omega/\tilde{c}_1)] \\ e_3 &= \exp[-ih_2(-\kappa+\omega/c_2)] \\ e_4 &= \exp[ih_2(\kappa+\omega/c_2)] \end{aligned} \tag{32}$$

$$s = z(1-i\varepsilon_1\omega^3)^{\frac{1}{2}}, \qquad z = \frac{c_1\rho_1}{c_2\rho_2}.$$

This determinant is expanded to yield

$$\cos(\omega\tilde{a}_1)\cos(\omega a_2) - \left(\frac{1+s^2}{2s}\right)\sin(\omega\tilde{a}_1)\sin(\omega a_2) = \cos(\kappa h) \tag{33}$$

where

$$\tilde{a}_1 = h_1/\tilde{c}_1 = a_1/(1-i\varepsilon_1\omega^3)^{\frac{1}{2}}, \qquad a_1 = h_1/c_1, \qquad a_2 = h_2/c_2, \tag{34}$$

and the total thickness is

$$h = h_1 + h_2. \tag{35}$$

This dispersion relation reduces properly for $p_1 = 0$ to the result obtained by Rytov [1] for no porosity. Also, if a_2 is zero, equation (33) reduces to

$$\kappa = \omega/\tilde{c}_1 \tag{36}$$

which is the dispersion equation for the porous medium considered in the preceding section.

A low-frequency expansion of equation (33) gives

$$\kappa = \omega/c_0 + \alpha\omega^3/c_0^2 + i\beta\omega^4 + \ldots \tag{37}$$

$$c_0^2 = \frac{h^2}{a_1^2 + 2ba_1a_2 + a_2^2}, \qquad b = \frac{1+z^2}{2z} \tag{38}$$

$$\alpha = \frac{c_0^3 a_1^2 a_2^2}{6h^2}\left(\frac{1-z^2}{2z}\right)^2 \tag{39}$$

$$\beta = \frac{\varepsilon_1 c_0}{2h^2}\left(a_1^2 + \frac{a_1 a_2}{z}\right) \tag{40}$$

It should be pointed out that a recent paper by McCoy [19] on the general subject of scattering in composites includes a low-frequency expansion for the wave number that has the same form as equation (37).

In Fig. 6, the variation is shown of c_0/v_0 with p_1 from equation (38) for $\nu_0 = 0{\cdot}3$, $h_2 = h_1$, $\rho_2 = 3\rho_0$, and several values of γ_0, which is defined as

$$\gamma_0 = \frac{\lambda_2 + 2\mu_2}{\lambda_0 + 2\mu_0}. \tag{41}$$

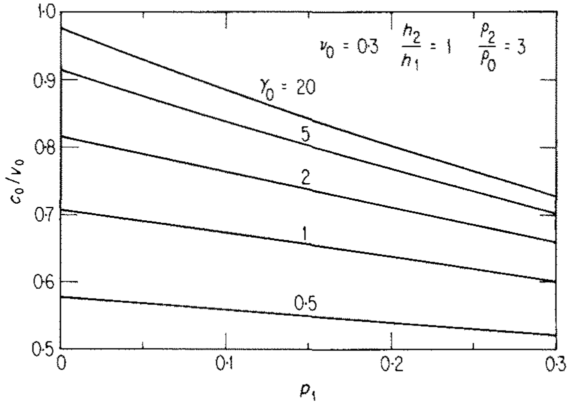

FIG. 6. Wave speed vs. porosity.

The composite wave speed decreases almost linearly as the porosity increases. The effect of porosity on the dispersion parameter α is shown in Fig. 7. Since $h_1 = h_2$, the total porosity in these examples is $p_1/2$; thus, 30 per cent porosity in constituent 1 corresponds to a total porosity of 15 per cent. The attenuation due to scattering as given by equation (40) is shown in Fig. 8 for $a = 0{\cdot}3h_1$.

It is possible to obtain dispersion and attenuation curves by determining numerically the values for κ_r and κ_i that satisfy equation (33) when ω is held constant. Figures 9 and 10 are included to show the effect of porosity on the frequency spectrum. As the porosity is increased, the real part of the lowest mode shifts as shown and the pass bands disappear since the imaginary part of the wave number is nonzero. Similar results were obtained by

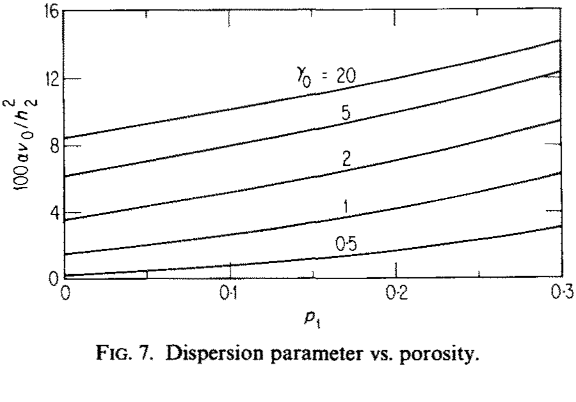

FIG. 7. Dispersion parameter vs. porosity.

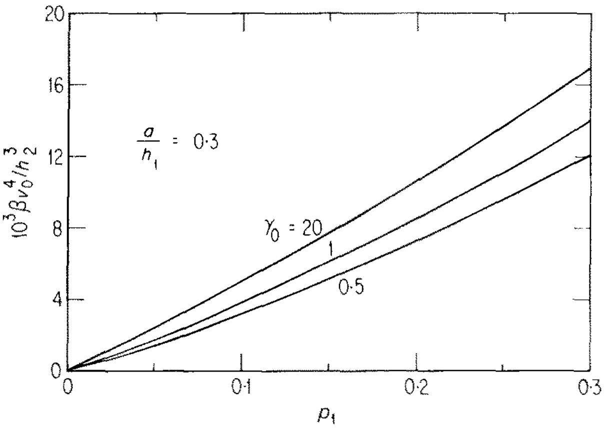

FIG. 8. Attenuation parameter vs. porosity.

the author [20] when thermoelastic coupling instead of porosity was included in the governing equations.

4. PULSE PROPAGATION IN POROUS COMPOSITES

The scattering of energy by voids or inclusions tends to damp out a step pulse. A composite material spreads out pulses. When the two effects are combined in a porous composite, the resulting pulse shape becomes complicated since the two effects are not cumulative. This is easily demonstrated by developing the solution to a Love–Rayleigh equation for composite material response that includes the dissipation term for Rayleigh scattering from voids. This approach should not be viewed as an exact treatment of pulse propagation in porous composites but as a technique for correcting the low-frequency dispersive Love–Rayleigh theory for decay due to scattering. This low-frequency approach does have merit since it has been shown experimentally by Whittier and Peck [21] that the

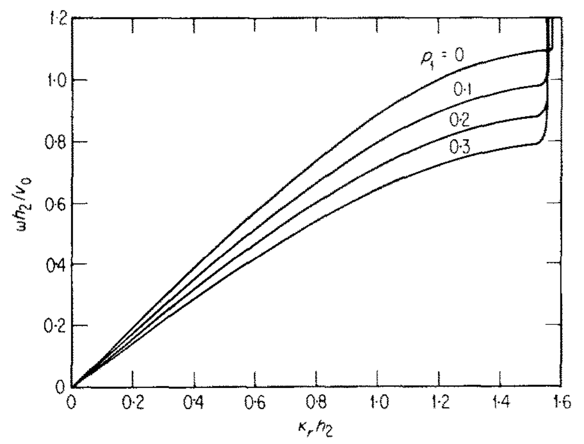

FIG. 9. Frequency vs. real part of wave number for porous composite.

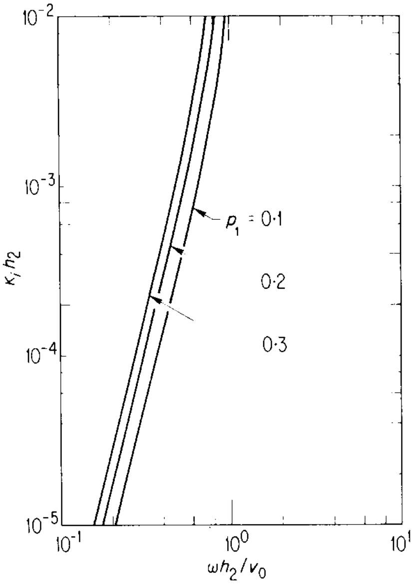

FIG. 10. Imaginary part of wave number vs. frequency for porous composite.

spatially-averaged far-field response of a nonporous laminated composite to a step pressure pulse can be predicted with a low-frequency analysis.

The same low-frequency relationship between the wave number and the frequency given in equation (37) is present in

$$u_{,xx} = \frac{1}{c_0^2}u_{,tt} - \frac{2\alpha}{c_0}u_{,xxtt} + 2c_0\beta u_{,xxttt}. \tag{42}$$

Assuming that the velocity is given at $x = 0$ as

$$u_{,t}(0, t) = H(t) \tag{43}$$

where $H(t)$ is the Heaviside function, Laplace transform techniques lead to

$$u_{,t} = \frac{1}{2\pi i}\int_{\mathrm{Br}_1} s^{-1}\exp[st - n(s)x]\,\mathrm{d}s \tag{44}$$

where Br_1 is the Bromwich contour in the right half s-plane and

$$n(s) = \pm\frac{s}{c_0}\left(1 + 2\frac{s^2\alpha}{c_0} - 2c_0\beta s^3\right)^{-\frac{1}{2}} \tag{45}$$

The sign of $n(s)$ is chosen to satisfy $Re[n(s)] > 0$ for convergence. The Br_2 contour, shown in Fig. 11 with the pole at zero and three branch points for $n(s)$, provides the inverted form

$$u_{,t} = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty \sin\left[\frac{r}{(2c_0\beta)^{\frac{1}{3}}}\left(t - \frac{x\cos\psi}{c_0(r_1r_2r_3)^{\frac{1}{2}}}\right)\right]\exp\left[\frac{rx\sin\psi}{(2c_0\beta)^{\frac{1}{3}}c_0(r_1r_2r_3)^{\frac{1}{2}}}\right]\frac{\mathrm{d}r}{r} \tag{46}$$

where

$$
\begin{aligned}
r_1 &= (r^2+b_1^2)^{\frac{1}{2}} \\
r_2 &= [b_2^2+(b_3-r)^2]^{\frac{1}{2}} \\
r_3 &= [b_2^2+(b_3+r)^2]^{\frac{1}{2}} \\
b_1 &= A+B+\frac{2\zeta}{3} \\
b_2 &= \frac{A+B}{2}-\frac{2\zeta}{3} \\
b_3 &= \frac{(A-B)}{2}\sqrt{3} \\
A &= \left[\frac{8\zeta^3}{27}+\frac{1}{2}+\left(\frac{8\zeta^3}{27}+\frac{1}{4}\right)^{\frac{1}{2}}\right]^{\frac{1}{3}} \\
B &= \left[\frac{8\zeta^3}{27}+\frac{1}{2}-\left(\frac{8\zeta^3}{27}+\frac{1}{4}\right)^{\frac{1}{2}}\right]^{\frac{1}{3}} \\
\phi_1 &= \pi-\tan^{-1}(r/b_1) \\
\phi_2 &= \cos^{-1}(b_2/r_2)\mathrm{SGN}(r-b_3) \\
\phi_3 &= \cos^{-1}(b_2/r_3) \\
\psi &= \frac{\pi}{2}-(\phi_1+\phi_2+\phi_3)/2 \\
\zeta &= \frac{\alpha}{c_0(2c_0\beta)^{\frac{1}{3}}}.
\end{aligned}
\tag{47}
$$

The expression for the velocity in equation (46) was evaluated for $x/h_2 = 100$, $h_2/h_1 = 1$, $\rho_2/\rho_0 = 3$, $\nu_0 = 0{\cdot}3$, $\gamma_0 = 20$, $p_1 = 0{\cdot}3$ and $a/h_1 = 0{\cdot}3$. It is compared with the corresponding nonporous case in Fig. 12. The addition of porosity in this example increases the risetime and period of oscillations and creates a small amount of damping. Arrival times are significantly different since c_0 decreases about 25 per cent for $p_1 = 0{\cdot}3$, as shown in Fig. 6.

5. CONCLUSIONS

A theoretical model for elastic wave propagation in porous composites has been presented that includes the geometric dispersion from the voids and the layering and the attenuation due to Rayleigh scattering by the voids. Dispersion and attenuation curves and a far-field pulse solution have been developed to indicate the influence of porosity on the dynamic response of porous composites. This analysis indicates that the interaction of random porosity and geometric dispersion in composite materials produces a complicated attenuation of pressure pulses. The porosity and periodicity of the composite combine to create dispersion and result in a frequency–dependent phase velocity

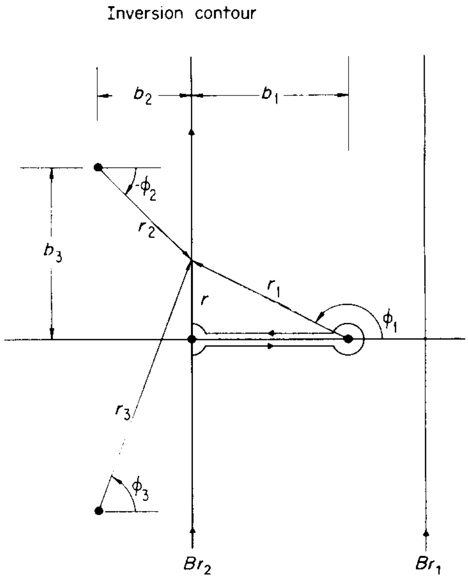

FIG. 11. Contour for inversion.

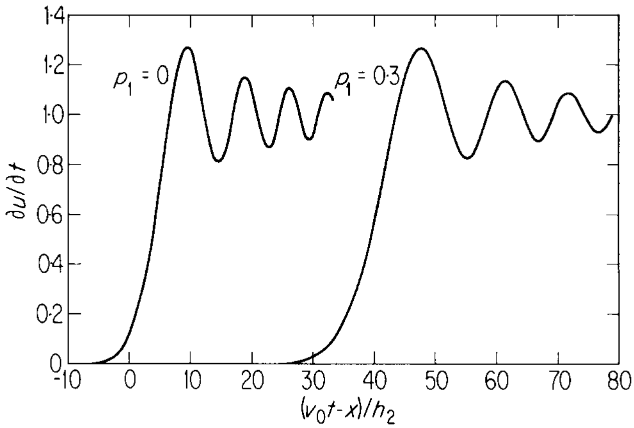

FIG. 12. Far field wave profiles.

that spreads out pulses. Porosity also involves scattering leading to additional attenuation and, if the pores are small and randomly located within a constituent of the composite, it may be possible to use a continuum approach for low-frequency response calculations.

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Абстракт—Дается анализ задачи распространения линейных упругих волн, нормальных к слоям периодически слоистых, пористых составных материалов. Пористость беспорядочно распределена повсюду одного компонента и составлена из лалых, сфероидальных пустот. Этот тип пористости назван беспорядочно периодическим. Он вызывает рэлеевское рассеяние, в котором длина ударяющей волны значительно больша, по сравнению с диаметром пустоты. Пористость уменьшает, также, скорость волны в компоненте и в связы с этом влияет на геометрическую дисперсию. Определяется диссипативное уравнеине движения для пористого материала, которое заключает скорость волны, в зависимости от пористости, и член рассеяния, дающий пространственное затухание. Затем применяется зто уравнение для компонента составного материала. Получаются зависимость дисперсии и решение для импульса, с целью указания значительности пористости в слоистых составных материалах. Делается вывод, что рэлеевское рассеяние дает малый эффект демпфирования в очертаниях отдаленных импульсов. Удовлетвориетльно можно моделировать пористость с лалыми пустотами с помошью эффективной скорости волны.